An map f analytic in a region A is *conformal* if $f'(z) \neq 0$ for all $z \in A$. By the chain rule, the composition of conformal maps is conformal. Conformal maps from regions A to B are often most easily found by considering a series of intermediate regions $A = A_1, \ldots, A_n = B$, corresponding 'standard' conformal maps $f_i : A_i \longrightarrow A_{i+1}$, and then taking the composition f. In questions, f is usually not a composition of more than 4 standard maps.

These standard maps include:

1. Möbius maps, e.g. $1/z$, $(z-1)/(z+1)$. The map $(z-a)/(z-b)$ is conformal except at b, and is extremely useful for mapping 'lozenges' - regions bounded by 2 circular arcs from a to b). $(z-1)/(z+1)$ maps the right half plane to the unit disc. Remember that Möbius maps send line segments and circular arcs to line segments and circular arcs, and since conformal maps send boundaries to boundaries, it is often straightforward to determine the image of a region bounded by line segments and circular arcs.

   Also, simple dilations λz, $\lambda \in \mathbb{R}$, and rotations $e^{i\theta} z$, $\theta \in \mathbb{R}$, are used all the time.

2. z^a is useful for mapping sectors of the plane to other sectors. E.g. z^3 maps the sector $0 < \arg(z) < \frac{\pi}{3}$ to $0 < \arg(z) < \pi$, i.e. the upper half plane. The principal branch of $z^{\frac{1}{2}}$ maps $-\pi < \arg(z) < \pi$, i.e. the plane cut along the negative real axis, to the right half plane. If a is not an integer then you have a multifunction, so make sure to define a cut and an appropriate analytic branch of the multifunction.

3. $\exp(z)$ is conformal everywhere and can map strips, either to sectors in the plane or annuli. E.g. $\exp(z)$ maps the horizonal strip $0 < \text{Im}(z) < \frac{\pi}{2}$ to the sector $0 < \arg(z) < \frac{\pi}{2}$, i.e. the top right quadrant. The vertical strip $0 < \text{Re}(z) < 1$ is mapped to the annulus $1 < |z| < \exp(1)$.

Being able to write a given region in more than one way is great for suggesting conformal maps. For example, the right half plane is given by

$$
\begin{aligned}
\{z \mid \text{Re}(z) > 0\} &= \{z \mid -\tfrac{\pi}{2} < \arg(z) < \tfrac{\pi}{2}\} \\
&= \{z \mid |z-1| < |z+1|\} \\
&= \{z \mid |(z-1)/(z+1)| < 1\}.
\end{aligned}
$$

The last representation suggests that the right half plane can be mapped onto the unit disc by $(z-1)/(z+1)$ and, by analysing points on the boundaries, this is indeed the case. The other half planes can be similarly represented.