Paper 3, Section I
1D Groups

Write down the matrix representing the following transformations of $\mathbb{R}^3$:

(i) clockwise rotation of $45°$ around the x axis,

(ii) reflection in the plane $x = y$,

(iii) the result of first doing (i) and then (ii).

Paper 3, Section I
2D Groups

Express the element $(123)(234)$ in S_5 as a product of disjoint cycles. Show that it is in A_5 . Write down the elements of its conjugacy class in A_5 .

Paper 3, Section II
5D Groups

(i) State the orbit-stabilizer theorem.

Let G be the group of rotations of the cube, X the set of faces. Identify the stabilizer of a face, and hence compute the order of G.

Describe the orbits of G on the set $X \times X$ of pairs of faces.

(ii) Define what it means for a subgroup N of G to be *normal*. Show that G has a normal subgroup of order 4.

Paper 3, Section II
6D Groups

State Lagrange's theorem. Let p be a prime number. Prove that every group of order p is cyclic. Prove that every abelian group of order p^2 is isomorphic to either $C_p \times C_p$ or C_{p^2}.

Show that D_{12}, the dihedral group of order 12, is not isomorphic to the alternating group A_4.

Paper 3, Section II
7D Groups

Let G be a group, X a set on which G acts transitively, B the stabilizer of a point $x \in X$.

Show that if $g \in G$ stabilizes the point $y \in X$, then there exists an $h \in G$ with $hgh^{-1} \in B$.

Let $G = SL_2(\mathbb{C})$, acting on $\mathbb{C} \cup \{\infty\}$ by Möbius transformations. Compute $B = G_\infty$, the stabilizer of ∞. Given

$$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G$$

compute the set of fixed points $\left\{ x \in \mathbb{C} \cup \{\infty\} \;\middle|\; gx = x \right\}$.

Show that every element of G is conjugate to an element of B.

Paper 3, Section II
8D Groups

Let G be a finite group, X the set of proper subgroups of G. Show that conjugation defines an action of G on X.

Let B be a proper subgroup of G. Show that the orbit of G on X containing B has size at most the index $|G : B|$. Show that there exists a $g \in G$ which is not conjugate to an element of B.